Hermite Polynominals With Two Independent Variables

Key Words : Heat equations, Hermite polynominals, zero sets

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1 Introduction

This note is a sequel of Watanabe[2]. Let W be the polynominal solution of the initial value problem for the Heat equation :

$$\begin{cases} \frac{\partial W}{\partial t} = \Delta W & \text{in} \quad \mathbf{R}^3, \\ W = p(x,y) & \text{on } t=0, \end{cases} \tag{1. 1}$$

where $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial}{\partial y^2}$ and p is a homogeneous polynominal of degree m with real coefficients. We put

$$H(x,y) := W(-1,x,y),\ H^*(x,y) := W(1,x,y) = (-i)^m H(i\,x,\,i\,y) \tag{1. 2}$$

where $i = \sqrt{-1}$. Then H satisfies the Hermite differential equation :

$$2\Delta H(x,y) - x\frac{\partial H(x,y)}{\partial x} - y\frac{\partial H(x,y)}{\partial y} + mH(x,y) = 0 \quad \text{in} \quad \mathbf{R}^2 \tag{1. 3}$$

and it can be written as follows.

$$H(x,y) = \sum_{k \geq 0} \frac{(-1)^k}{k!} \Delta^k p(x,y). \tag{1. 4}$$

We say that a polynominal solution, H, with real coefficients of Hermite differential equation (1. 3) is a Hermite polynominal and that H^*, which is defined by (1. 2), is its conjugate Hermite polynominal.

The aim of this note is to study zero sets of such polynominals and we shall apply results, obtained here, in order to analyise zero sets of solutions of second order parabolic partial differential equations with two space-dimension.

Watanabe[3] traited such study in the case of one space-dimension and he determinated minutely the local natures of zero sets of non analytic solutions under two points boundary values conditions. One of differences between the case of space-dimension 1 and 2 is the following. Even if the initial date has singular points, the toplogical natures of zero sets of solutions at the past and at the future does not necessarily change.

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2 Zero points at the infinity

Let H be a non zero Hermite polynominal. It is well known that, in a small neigbourhood of a point P belonging to the singular part, denoted by $S(H)$, of its zero set, denoted by $Z(H)$:

$$\begin{aligned} Z(H) &= \{(x,y) \in \mathbf{R}^2\,;\, H(x,y) = 0\}, \\ S(H) &= \{(x,y) \in Z(H)\,;\, H_x(x,y) = H_y(x,y) = 0\}, \end{aligned}$$

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the set $Z(H)$ is locally equal to the union of $\nu(H, P)$ −analytic curves passing through P at which forms an equiangular system. Here $\nu(H, P)$ is the vanishing order of H at P.

In this section we shall study zero sets of Hermite polynominals at the infnity. At frist we prepare three families of polynominals. Let

$$H_m(x) = \prod_{j=1}^{m} (x - \tau(m, j)) \tag{2.1}$$

be the Hermite polynominal of order m with one variable where we enumerate its zero points so that

$$\tau(m, 1) < \tau(m, 2) < \cdots < \tau(m, m) \tag{2.2}$$

and hence

$$W_m(t, x) = \begin{cases} \prod_{j=1}^{l} (x^2 + \tau(m, j)^2 t) & \text{when } m = 2l, \\ x \prod_{j=1}^{l} (x^2 + \tau(m, j)^2 t) & \text{when } m = 2l+1, \end{cases} \tag{2.3}$$

is the polynominal solution of the Heat equation with $W_m(-1, x) = H_m(x)$.

For $m/2 \geq d \geq 1$, put

$$V_{m,d}(t, r) = \sum_{k=0}^{d} \frac{t^k}{k!} \frac{d!\,(m-d)!}{(d-k)!\,(m-d-k)!} r^{d-k}, \tag{2.4}$$

then it saisfies

$$r^{m-2d} \frac{\partial}{\partial t} V = \frac{\partial}{\partial r} \{r^{m-2d+1} \frac{\partial V}{\partial r}\}.$$

Putting

$$u_{m,d}(s) = \exp\left\{\frac{1}{2} B_{m,d}(s)\right\} \sum_{k=0}^{d} \frac{s^{d-k}}{k!\,(d-k)!\,(m-d-k)!},$$

$$B_{m,d}(s) = s + (m-2d+1)\log|s|,$$

$$q_{m,d}(s) = \frac{1}{4} + \frac{m+1}{2s} + \frac{(m-2d+1)(m-2d-1)}{4s^2},$$

we have

$$u''_{m,d}(s) = q_{m,d}(s)\, u_{m,d}(s)$$

and so by Strum's comparasion theorm we have the following.

Lemma 2.1. *For each m, d with $m/2 \geq d \geq 1$, $u_{m,d}$ has d-simple zero points $\tau(m, d, j)$, $1 \leq j \leq d$, in $s < 0$ and*

$$V_{m,d}(t, r) = \prod_{j=1}^{d} (r - \tau(m, d, j)t).$$

Since

$$\frac{\partial}{\partial r} V_{m,d}(t, r) = d\, V_{m-1,d-1}(t, r)$$

it is easy to see that

$$\tau(m, d, i) \neq \tau(m-1, d-1, l) \tag{2.5}$$

for any $m/2 \geq d \geq 2$, $1 \leq i \leq d$, $1 \leq l \leq d-1$.

The last family $\{R_{m,n}(t,r)\ ;\ m-n$ is even and $0 \le n \le m\}$ is defined by

$$R_{m,n}(t,r) = r^m + \sum_{0<2l\le m-n} \frac{t^l}{l!} \left\{ \prod_{k=0}^{l-1} ((m-2k)^2 - n^2) \right\} r^{m-2l}. \tag{2.6}$$

Then the polynominal solution W of (1.1) with $W|_{t=0} = r^m \cos n\theta$ is equal to $R_{m,n}(t,r) \cos n\theta$ where we used the polar coordinates $x = r\cos\theta$, $y = r\sin\theta$. By the analogous arguments to the family $\{V_{m,d}\}$, we have that for some $\mu_k(m,n) < 0$

$$R_{m,n}(t,r) = r^n \prod_{k=1}^{(m-n)/2} (r^2 - \mu_k(m,n)t).$$

Now we are ready to state the main theorem in this section. Let p be a homogeneous polynominal of degree m with real coefficients and we put

$$p(x,y) = \prod_{j=1}^{d} (y - \lambda_j x)^{d_j} \tag{2.7}$$

where $\lambda_j \neq \lambda_k$ for $j \neq k$. We use the notation : $\delta_j = \sqrt{1+\lambda_j^2}$.

Theorem 2.2. Let H be a Hermite polynominal given by (1.4) with (2.7). Then there are holomorphic functions, $\psi_{j,l}$, $1 \le j \le d$, $1 \le l \le d_j$, *in some neigbourhood of the origin in* $\mathbf{C}$ *such that* $\psi_{j,l}(0) = 0$ *and*

$$H(x,y) = \prod_{j=1}^{d} \prod_{l=1}^{d_j} (y - \phi_{j,l}(x))$$

where $\phi_{j,l}$ *are given by the following.*

$$\phi_{j,l}(x) = \begin{cases} \lambda_j x + \tau(d_j, l)\delta_j + \psi_{j,l}(1/x) & \text{when } \lambda_j \neq \pm i, \\ \lambda_j x + \{2\tau(m, d_j, l)\lambda_j + \psi_{j,l}(1/x)\}/x & \text{when } \lambda_j = \pm i. \end{cases}$$

Proof. For fixed j, put $\eta = y - \lambda_j x$, $\xi = x$ and

$$p(x,y) = \sum_{k=0}^{m-d_j} p_k \xi^{m-d_j-k} \eta^{k+d_j}, \quad p_0 \neq 0.$$

Suppose that $\lambda_j \neq \pm i$. Then we have

$$W(-t^2, x, y) = p_0 \xi^{m-d_j} W_{d_j}(-\delta_j^2 t^2, \eta) + R(t, \xi, \eta)$$

where W_{d_j} is given by (2.3) and R can be written, for some constants $C_{a,b,c}$, as follows.

$$R(t,\xi,\eta) = \sum_{a+b+c=m>a+d_j} C_{a,b,c} \xi^a \eta^b t^c.$$

From this

$$H(x,y) = p_0 \xi^{m-d_j} \left\{ \prod_{k=1}^{d_j} (\eta - \tau(d_j,k)\delta_j) + \sum_{a+b+c=m>a+d_j} \frac{C_{a,b,c}}{p_0} \eta^b (1/\xi)^{m-a-d_j} \right\}$$

and then for sufficiently large $|\xi|$ the equation $H=0$ has roots of the form :

$$\eta = \tau(d_j,k)\delta_j + \psi_{j,k}(1/\xi), \quad 1 \le k \le d_j,$$

where $\psi_{j,k}$ are holomorphic in some neigbourhood of the origin with $\psi_{j,k}(0)=0$.

Suppose that $\lambda_j=\pm i$. By analoguous arguments used above, it follows from

$$\sum_{k=0}^{d_j}\frac{(-1)^k}{k!}\left\{-2\lambda_j\frac{\partial^2}{\partial\xi\partial\eta}\right\}^k\xi^{m-d_j}\eta^{d_j}=\xi^{m-2d_j}V_{m,d_j}(2\lambda_j,\xi\eta)$$

that for some constants $C_{a,b}$

$$H(x,y)=p_0\xi^{m-2d_j}\left\{V_{m,d_j}(2\lambda_j,\xi\eta)+\sum_{b>0}C_{a,b}(\xi\eta)^a(1/\xi)^b\right\}$$

and so Lemma 2.1 completes the proof.

Corollay 2.3. *Under the notation in Theorem 2.2, we have the following.*

(1) $\phi_{j,l}(x)$ *is analytic in* x *if and only if* λ_j *is real.*

(2) $i\phi_{j,l}(ix)$ *is analytic in* x *if and only if* λ_j *is real,* d_j *is odd and* $l=(1+d_j)/2$.

(3) *The conjugate Hermite polynominal* H^* *of* H *can be written as follows.*

$$H^*(x,y)=\prod_{j=1}^{d}\prod_{l=1}^{d_j}(y+i\phi_{j,l}(ix)).$$

Proof. Since H has real coefficients,

$$H(x,y)=\prod_{j=1}^{d}\prod_{l=1}^{d_j}(y-\overline{\phi_{j,l}(\overline{x})})$$

which implies, from behaviors at the infinity of $\phi_{j,l}$, that $\phi_{j,l}(x)=\overline{\phi_{j,l}(\overline{x})}$ if and only if λ_j is real.

It follows from (1.2) that the assertion (3) holds and so $\phi_{j,l}(ix)=-\overline{\phi_{j,l}(\overline{ix})}$ if and only if λ_j is real and $\tau(d_j,l)=0$, i.e. d_j is odd and $l=(1+d_j)/2$.

3 Critical points

In this section we consider the sets of critical points of Hermite polynominals H:

$$\Sigma(H)=\{(x,y)\in\mathbf{R}^2\,;\,H_x(x,y)=H_y(x,y)=0\}$$

and that of conjugate Hermite polynominals.

Theorem 3.1.

(1) *Let* H *be a non constant Hermite polynominal such that the dimension of the set* $\Sigma(H)$ *is equal to* 1. *Then* $S(H)$ *is empty and* H *satifies one of the following conditions.*

(1-1) *After a rotation around the origin,* H *is a polynominal with one variable.*

(1-2) H *is a polynominal of* $r=\sqrt{|x|^2+|y|^2}$.

(2) *The dimension of* $\Sigma(H^*)$ *is equal to* 1 *if and only if* H^* *is a polynominal with one variable of even order, after a rotation around the origin.*

Proof. quad At frist we show (1). Let H be given by (1.4) with (2.7). Choosing a rotation around the origin, we may assume that $\sum_{j=1}^{d}d_j\lambda_j\neq 0$. So the assumption means that the resultant as polynominals in y of H_x and H_y, identitcally vanishes in $\mathbf{C}$. It follows from Theorem 2.2 that the common zero set of H_x and H_y contains a curve Γ of the form : $y=\sigma x+\phi(x), |x|>>1$ such that the following holds.

(*i*). When $\sigma\neq\pm\sqrt{-1}$, $\phi(x)=\tau(a,l)\sqrt{1+\sigma^2}+O(|x|^{-1})$ as $|x|\to\infty$.

(*ii*). When $\sigma=\pm\sqrt{-1}$, $\phi(x)=2\tau(m-1,a,l)\ \sigma x^{-1}+O(|x|^{-2})$ as $|x|\to\infty$.

Here we used the notation in Theorem 2.2 and a is the multiplicity at zero $(1,\sigma)$ of p_y and $1\leq l\leq a$.

Since $(1, \sigma)$ is a common zero of p_x and p_y, it is also of p.

Suppose that $\sigma \neq \pm\sqrt{-1}$. Then we have that as $|x| \to \infty$ on Γ

$$H(x,y) = \prod^{(1)} \prod_{k=1}^{d_j} \{(\sigma - \lambda_j)x + O(1)\}$$
$$\times \prod_{k=1}^{a+1} \left\{ (\tau(a,l) - \tau(a+1,k)) \sqrt{1+\sigma^2} + O(|x|^{-1}) \right\}.$$

Here $\prod^{(1)}$ means the product over j such that $\lambda_j \neq \sigma$. From this and the fact that H is non zero constant on Γ, we have $m = a+1$. This means that H is a Hermite polynominal with one variable.

Suppose that $\sigma = \pm\sqrt{-1}$. Then we have the following.

$$H(x,y) = \prod^{(1)} \prod_{k=1}^{d_j} \{(\sigma - \lambda_j)x + O(1)\}$$
$$\times \prod_{k=1}^{a+1} \{ (\tau(m-1,a,l) - \tau(m,a+1,k)) 2\sigma/x + O(|x|^{-2}) \}.$$

By the same arguments we obtain that $a+1 = m-a-1$ and hence $p = r^{2(a+1)}$.

When H is a Hermite polynominal with one variable, it is well known that $S(H)$ is empty. When H is a polynominal of r, $H(x,y) = R_{m,0}(-4, r^2)$, which is given by (2.6), and so by Lemma 2.1 $S(H)$ is empty.

By the same arguments we have that dim $\Sigma(H^*) = 1$ implies that H^* is equal to either a function with one variable, after a rotation around the origin, or a function of r. In the former case it must be of even order and in the latter case $\Sigma(H^*)$ is equal to $\{(0,0)\}$.

Lemma 3.2. *Let H be a Hermite polynominal, given by (1.4) with (2.7). When the dimension of $\Sigma(H)$ is equal to 0, then this set is finite. Moreover*

$$\liminf_{|x|+|y| \to \infty} \frac{|\operatorname{grad} H(x,y)|}{(|x|+|y|)^{m-d^*-1}} > 0. \tag{3.1}$$

Here

$$d^* = \begin{cases} 0, & \text{when } Z(p_x) \cap Z(p_y) \cap \mathbf{R}^2 = \{(0,0)\}, \\ \max\{d_j \,;\, \lambda_j \text{ is real}\}, & \text{when } Z(p_x) \cap Z(p_y) \cap \mathbf{R}^2 \neq \{(0,0)\}. \end{cases} \tag{3.1}$$

Proof. We note, by virtue of the theorem of Whiteny (see for example, Milnor[11]), that any algebraic set of dimension 0 is finite.

When $d^* = 0$, it is easy to see (3.1).

We assume that $d^* > 0$ and $\sum_{j=1}^{d} d_j \lambda_j \neq 0$.

Let $(1, \lambda)$ be in $Z(p) \cap R^2$ with multiplicity $a \geq 2$ and let $\phi^{(j)}(x)$, $j = 1, 2$ be analytic functions near the infinity where

$$\begin{cases} H_x(x, \phi^{(1)}(x)) = H_y(x, \phi^{(2)}(x)) = 0, \\ \phi^{(j)}(x) = \lambda x + \tau(a-1,l)\sqrt{1+\lambda^2} + O(1/|x|), \end{cases} \tag{3.2}$$

for some $l \leq a-1$. By assumption and Theorem 3.1, we obtain that $a < m$ and that $\phi^{(1)} - \phi^{(2)}$ does not identitically vanish and so there is a constant $\sigma \leq -1$ such that for large $|x|$

$$C_1|x|^\sigma \geq |\phi^{(1)}(x) - \phi^{(2)}(x)| \geq C_2|x|^\sigma.$$

Here C_1 and C_2 are positive constants and we will denote by C_j positive constants. We claim that $\sigma = -1$.

For fixed x_o we have

$$
\begin{aligned}
&|H(x,\phi^{(1)}(x))| - |H(x_o,\phi^{(1)}(x_o))| \\
&\le \left| \int_{x_o}^{x} \frac{d}{dx} H(x,\phi^{(1)}(x))\,dx \right| \le C_3 \left| \int_{x_o}^{x} |x|^{m-a+\sigma}\,dx \right| \\
&\le C_4 \begin{cases} |x|^{m-a+c+1} + C_5, & \text{when } m-a+\sigma \neq -1, \\ \log|x| + C_5, & \text{when } m-a+\sigma = -1, \end{cases}
\end{aligned}
$$

and using (2. 5)

$$
|H(x,\phi^{(1)}(x))| \ \ge\ C_6\,|x|^{m-a} \prod_{k=1}^{a} |\phi^{(1)}(x) - \lambda x - \tau(a,k)\sqrt{1+\lambda^2} - O(1/|x|)| \ge C_7\,|x|^{m-a}.
$$

Combinating two inequalities stated above, we have $\sigma = -1$.

Suppose that there is a sequence, $\{(x_n, y_n)\}_{n=1}^{\infty}$, in R^2 such that

$$
\lim_{n\to\infty} |x_n| = \infty, \quad \lim_{n\to\infty} \frac{|\operatorname{grad} H(x_n,y_n)|}{(|x_n|+|y_n|)^{m-d^*-1}} = 0.
$$

Then there is, by taking subsequence if necessary, one and only one $(1,\lambda)$ in $Z(p_x)\cap Z(p_y)\cap R^2$ such that

$$
\lim_{n\to\infty} \frac{y_n - \lambda x_n}{|x_n|+|y_n|} = 0.
$$

On the other hand we obtain, denoting by a the multiplicity at $(1,\lambda)$ of p,

$$
\frac{|\operatorname{grad} H(x_n,y_n)|}{(|x_n|+|y_n|)^{m-d^*-1}} \ge C_8 |x_n|^{d^*-a+1} \prod_{j=1}^{a-1} |y_n - \lambda x_n - \tau(a-1,j)\sqrt{1+\lambda^2} + O(1/|x_n|)|,
$$

which implies $y_n - \lambda x_n - \tau(a-1, l)\sqrt{1+\lambda^2} \to 0$ as $n\to\infty$ for some l and hence we have, using $\phi^{(j)}$, $j=1, 2$, verifying (3. 2),

$$
\frac{|\operatorname{grad} H(x_n,y_n)|}{(|x_n|+|y_n|)^{m-d^*-1}} \ge C_9\,|x_n|^{d^*-a+1}\{|y_n - \phi^{(1)}(x_n)| + |y_n - \phi^{(2)}(x_n)|\} \ge C_{10}|x_n|^{d^*-a}.
$$

Consequently we find a contradiction and thus we have (3. 1).

Remark 3. 3. *We have also the following estimations for any Hermite polynominals given by* (1. 4) *with* (2. 7). *When* $d^* \ge 2$,

$$
\liminf_{|x|+|y|\to\infty} \frac{|H(x,y)| + |H_y(x,y)|}{(|x|+|y|)^{m-d^*}} > 0. \tag{3. 3}
$$

The same estimations as (3. 1) *and as* (3. 3) *for conjugate Hermite polynominals hold.*

4 Nodal domains

In this section we consider nodal domains of Hermite polynominal H, given by (1. 4) with (2. 7) and its conjugate polynominals H^*.

We use the following notation. For a subset A of R^2 we denote by $N(A)$, $N_c(A)$ the number of components of A, that of compact components of A, respectively.

For a polynominal p with (2. 7) we put

$m(p, +0)$ = the number of j such that λ_j is real and d_j is odd,

$$m(p,\,0) \;=\; \text{the number of } j \text{ such that } \lambda_j \text{ is real},$$
$$m(p,\,-0) \;=\; \sum\{\,d_j\,;\,\lambda_j \text{ is real}\,\}.$$

Proposition 4. 1.
(i). $N(\mathrm{R}^2 \setminus Z(H^*)) \leq 2m(p,+0) \leq N(\mathrm{R}^2 \setminus Z(p))$ *and*

$$\sum\{\,\nu(H^*,\,P)-1\,;\,P\in S(H^*)\,\} \;\leq\; m(p,\,+0)-1.$$

(ii). $Z(p)$ *is homeomorphic to* $Z(H^*)$ *if and only if*

$$m(p,\,0)=m(p,\,+0)=\nu(H^*,\,(0,0))\geq 1. \tag{4.1}$$

Moreover when it is so, $N(\mathrm{R}^2 \setminus Z(H)) \geq N(\mathrm{R}^2 \setminus Z(p))$.
(iii). $Z(p)$ *is homeomorphic to* $Z(H)$ *if and only if*

$$\begin{cases} S(H)\setminus\{\,(0,0)\,\} \text{ is empty and} \\ m(p,\,0)=m(p,\,+0)=\nu(H,\,(0,0))\geq 1=N(Z(H)). \end{cases} \tag{4.2}$$

Moreover when it is so, $Z(p)$ *is homeomorphic to* $Z(H^*)$.

Proof. We use the fact that H^* has no bounded nodal domain, which follows from the maximum principle. Using this fact, *(i)* is a consequence of Corollary 2. 3 and

$$N(\mathbf{R}^2\setminus Z(H^*))=1+\sum\{\,\nu(H^*,\,P)-1\,;\,P\in S(H^*)\,\}+m(p,\,+0)\;\leq\;2m(p,\,+0).$$

Suppose that $Z(p)$ is homeomorphic to $Z(H^*)$. Then Corollary 2. 3 implies $m(p,+0)=m(p,0)$, noted by n. When $n=0$, $Z(H^*)$ is empty. When $n=1$, $Z(H^*)\ni(0,0)$ and $S(H^*)$ is empty. When $n\geq 2$, $\nu(H^*,P)=n$ for some $P\in S(H^*)$ and by virtue of the assertion *(i)* we obtain $S(H^*)=\{P\}$ and so $P=(0,0)$.

Conversely, suppose that (4. 1) holds. When $n=1$, it follows from Corollary 2. 3 that $Z(H^*)$ is a 1-dimensional non singular curve. Suppose that $n\geq 2$. Since the number of components of $Z(H^*)\setminus\{(0,0)\}$, whose closure contains $(0,0)$, is equal to $2n$ and such components are unbounded, the union of such components is equal to $Z(H^*)\setminus\{(0,0)\}$ and $S(H^*)=\{(0,0)\}$.

$$\begin{aligned} &N(\mathbf{R}^2\setminus Z(H)) \\ =\;& 1+N_c(Z(H))+\sum\{\,\nu(H,\,P)-1\,;\,P\in S(H)\,\}+m(p,\,-0) \\ \geq\;& \nu(H,\,(0,0))+m(p,\,-0)\geq N(\mathbf{R}^2\setminus Z(p)). \end{aligned}$$

(iii). Suppose that $Z(p)$ is homeomorphic to $Z(H)$. By the same arguments for *(ii)*, we have (4. 2).

Conversely suppose that (4. 2) holds. Then each component of $Z(H)\setminus\{(0,0)\}$ is unbounded. If not, by Theorem 2. 2 there is an unbounded analytic curve, not containing $(0,0)$, which is a contradiction counter $N(Z(H))=1$. Hence $Z(H)\setminus\{(0,0)\}$ is the union of such components and so $Z(p)$ is homeomorphic to $Z(H)$. It is clear that (4. 2) implies (4. 1).

Example 4. 2. *There is non harmonic Hermite poylnominal H, which is given by* (1. 4), *such that $Z(H)$ is homeomorphic to $Z(p)$. One of them is given by*

$$p(x,y)=x\,\{\,(y-x)^2+x^2\,\}.$$

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2変数エルミート多項式

キーワード：熱方程式，エルミート多項式，零点集合

渡 辺 金 治

熱方程式に代表される放物型偏微分方程式の解の零点集合を解析する上で重要な役割をはたす，エルミート多項式およびそれに付随する多項式について，それらの零点集合を中心に考察する。